

**AD-A246 349**



ARL-STRUC-TM-527

AR-006-091

②



**DEPARTMENT OF DEFENCE**  
**DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION**  
**AERONAUTICAL RESEARCH LABORATORY**  
**MELBOURNE, VICTORIA**

Aircraft Structures Technical Memorandum 527

**AN OUTLINE OF A NUMERICAL SCHEME FOR CALCULATING TWO-  
DIMENSIONAL TIME LINEARISED TRANSONIC FLOW USING THE GREEN'S  
FUNCTION METHOD**

by

I. H. Grundy

**DTIC**  
**ELECTE**  
**FEB 26 1992**  
**S D D**

This document has been approved  
for public release and sale; its  
distribution is unlimited.

Approved for public release

© COMMONWEALTH OF AUSTRALIA 1991

AUGUST 1991

**This work is copyright. Apart from any fair dealing for the purpose of study, research, criticism or review, as permitted under the Copyright Act, no part may be reproduced by any process without written permission. Copyright is the responsibility of the Director Publishing and Marketing, AGPS. Enquiries should be directed to the Manager, AGPS Press, Australian Government Publishing Service, GPO Box 84, CANBERRA ACT 2601.**

AR-006-091

DEPARTMENT OF DEFENCE  
DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION  
AERONAUTICAL RESEARCH LABORATORY

Aircraft Structures Technical Memorandum 527

AN OUTLINE OF A NUMERICAL SCHEME FOR CALCULATING TWO-  
DIMENSIONAL TIME-LINEARISED TRANSONIC FLOW USING THE GREEN'S  
FUNCTION METHOD

by

I. H. Grundy

SUMMARY

*A numerical scheme is outlined for the calculation of two dimensional time-linearised transonic flow about an aerofoil or collection of components, using the Green's function method. Computed results are presented for various configurations for the simpler subproblem of steady subsonic flow.*



© COMMONWEALTH OF AUSTRALIA 1991

A section For	
NTIS	CRA&I
DTIC	TAB
Unpublished	
Justification	
By	
Distribution	
Availability Codes	
Dist.	Avail and/or Special
A-1	

POSTAL ADDRESS: Director, Aeronautical Research Laboratory  
506 Lorimer Street, Fishermens Bend Victoria 3207  
Australia

92 2 20 047

92-04510  
■■■■■■■■■■

## CONTENTS

1. INTRODUCTION . . . . .	1
2. TRANSONIC PROBLEM . . . . .	1
3. PROPOSED NUMERICAL SCHEME - AN OUTLINE . . . . .	4
4. SUBSONIC PROBLEM . . . . .	5
5. RESULTS AND DISCUSSION . . . . .	8
6. CONCLUSION . . . . .	8
REFERENCES . . . . .	9
FIGURES 1-5 . . . . .	11
DISTRIBUTION	
DOCUMENT CONTROL DATA	

## 1. INTRODUCTION

This article presents an outline of a numerical scheme for calculating two-dimensional unsteady transonic flow about an aerofoil, or a collection of aerofoil components. The aerofoil components are assumed to be thin, and the effects of viscosity are neglected. Hence, the flow is assumed to be governed by the transonic small disturbance equation [1], [7].

The approach followed here is the Green's function (integral equation) method. This method, see [8-10] and also [2], was first introduced to compute unsteady subsonic aerodynamics in three dimensions. However, the method was extended in [14] to also handle transonic aerodynamics (see also [3]). The main advantage of the method is that it is easy to implement for complex geometries, since body-fitting grid systems are not required [5], [15].

In [14] and [3], the Green's function method involves "time-integration". That is, the flow about the aerofoil is calculated at each moment as the program steps forward in time. Alternatively, significant savings in computer time can be made by "time-linearising", i.e. by assuming that the aerofoil oscillates harmonically about some mean steady position ([5], [8], [9], and [11]).

The cost of "time-linearisation" is an inability to model highly non-linear phenomena such as shock appearance and disappearance, and large scale shock motion, such as that identified in [13]. However, the method is compatible with conventional subsonic (linear) flutter analysis, and significantly easier to program than "time-integration" methods.

As a first step towards a full "time-linearised" transonic capability, results are presented here for a simple sub-problem of two-dimensional unsteady transonic flow, namely the problem of two-dimensional steady subsonic flow. This preliminary problem, for which analytical solutions are available, provides a useful initial check on the method's accuracy and a pointer to possible problems in the full transonic formulation.

## 2. TRANSONIC PROBLEM

Under the assumptions that the fluid is isentropic and inviscid, the flow is irrotational and the aerofoil is thin and undergoes small unsteady motion, there exists a perturbation potential  $\phi$  which must satisfy the unsteady small perturbation transonic equation [1], [7], namely

$$[(1 - M^2) - M^2(1 + \gamma)\phi_x]\phi_{xx} + \phi_{yy} - 2M^2\phi_{xt} - M^2\phi_{tt} = 0. \quad (1)$$

where  $\gamma$  is the ratio of specific heats and the  $x$ -axis is in the streamwise direction. The Mach number  $M$  is assumed to be near, but less than unity, meaning that only "sub"-transonic flow will be considered here.

In order to reduce the computational time and complexity of the formulation, the governing equation is "time-linearised" as in [5] and [11]. That is, the potential is separated into a steady part and a small harmonically oscillating unsteady part. This leads to a non-linear boundary value problem for the

steady potential and a linear boundary value problem for unsteady potential (which is linked to the steady problem).

Equation (1) is "time-linearised" by setting  $\phi = \phi_s + \phi_u$ , where  $\phi_u \ll \phi_s$ , and keeping only the higher order terms. Accordingly, the steady potential  $\phi_s$  must satisfy

$$(1 - M^2)\phi_{sxx} + \phi_{syy} = M^2(1 + \gamma)\phi_{sx}\phi_{sxx} \quad (2)$$

which can be rewritten, using the Prandtl-Glauert transformation  $X = x$ ,  $Y = \beta y$  (setting  $\beta = \sqrt{1 - M^2}$ ), as

$$\phi_{sXX} + \phi_{sYY} = \sigma_s, \quad (3)$$

where the non-linear term  $\sigma_s$  is given by

$$\sigma_s = \frac{M^2(1 + \gamma)}{1 - M^2} \phi_{sX} \phi_{sXX}. \quad (4)$$

The unsteady potential  $\phi_u$  must then satisfy the linear equation

$$(1 - M^2)\phi_{uxx} + \phi_{uyy} - 2M^2\phi_{uxt} - M^2\phi_{utt} = M^2(1 + \gamma)\frac{\partial}{\partial x}[\phi_{ux}\phi_{sx}]. \quad (5)$$

Application of the above Prandtl-Glauert transformation, plus the further substitution (assuming harmonic time dependence) of

$$\phi_u = \Phi_u e^{ik(t + M^2 X/\beta^2)} \quad (6)$$

into (5) leads to an equation for the new dependent variable  $\Phi_u$ , namely

$$\Phi_{uXX} + \Phi_{uYY} + h^2\Phi_u = \sigma_u, \quad (7)$$

where  $h = kM/\beta$  and the  $\sigma_u$  term on the right hand side (which is linear but dependent on the steady potential) is given by

$$\sigma_u = \frac{M^2(1 + \gamma)}{1 - M^2} \frac{\partial}{\partial X}[\phi_{sX} \frac{\partial}{\partial X}(\Phi_u e^{ikM^2 X/\beta^2})]. \quad (8)$$

It is not strictly necessary to use equations (3) and (4) to calculate the underlying steady flow field. An alternative, see [4] and [5] and elsewhere, is to use the steady solution from a much simpler formulation than the transonic small perturbation equation, thus making computer time savings.

Conversely, the steady flow field could be generated by a more complicated equation such as the full potential equation. The advantage of this approach over the use of the small disturbance equation is that shock strength and position are modelled more realistically.

An attractive feature of using the full-potential equation for the steady flow field is that this equation can be solved with relatively little extra cost over the small disturbance equation. Solving the steady full potential equation simply means using a different non-linear term  $\sigma_s$ , as in [15].

The method of solution of both the steady and unsteady problems is based on the Morino integral equation method [8], in which Green's Formula is used to construct an integral equation for the velocity potential  $\phi$ , where  $\phi$  represents  $\phi_s$  in the steady case, and  $\Phi_u$  in the unsteady case.

Green's formula for both the steady and unsteady equations relates the potential  $\phi_P$  at a point  $P$  in the flow domain to an integral of the potential over the aerofoil boundary  $C_S$ , the wake contour  $C_W$ , any shock discontinuities  $C_D$ , and the contour at infinity  $C_\infty$ . It has the form

$$E\phi_P = \int_C \left( G \frac{\partial \phi}{\partial N} - \phi \frac{\partial G}{\partial N} \right) d\ell + \int_F G \sigma dS \quad (9)$$

where  $C = C_S \cup C_W \cup C_D \cup C_\infty$  (as shown in Figure 1),  $N$  is the inward normal to the fluid in the Prandtl-Glauert space, and  $\sigma$  represents either  $\sigma_s$  or  $\sigma_u$ .

The domain function  $E$  is defined by

$$E = \begin{cases} 1, & \phi_P \text{ within flow field} \\ \frac{1}{2}, & \phi_P \text{ on the boundary} \\ 0, & \phi_P \text{ outside flow field.} \end{cases} \quad (10)$$

For the steady problem

$$G = \frac{1}{2\pi} \log R \quad (11)$$

(where  $R$  is the distance to the singularity) is an appropriate Green's function, while for the unsteady problem

$$G = \frac{1}{4} i H_0^{(2)}(hR) \quad (12)$$

(where  $H_0^{(2)}$  is the Hankel function of the second kind of order zero) is an appropriate Green's function.

Using the far-field expansions in [1] and [6], and the Green's functions above, it can be shown that at infinity the contribution to the righthand side of (9) is at most a constant. Hence, the boundary integral over  $C_\infty$  may be omitted. Also, the integral over the shock discontinuity  $C_D$  can be combined with the volume integral in such a way that there is no explicit contribution to  $\phi_P$  (as a result of the shock jump conditions) [6], [14]. Since the new integral equation generated this way can be shown to differ very little from the original integral equation "at the numerical algorithm level" [14] when discretised, the original equation can be used ignoring altogether the presence of shocks, which are said to be "captured". As in [12], artificial viscosity can be used when evaluating the field integrals to guarantee the absence of (i.e. smooth out) shock waves.

A primary advantage of the integral equation approach is that no complex, body-fitting grid generation scheme is needed within the flow field (such as would be required in a finite difference method). Most of the effort in grid generation is needed at the aerofoil boundary, which is approximated by small elements (line segments in 2D).

Although the flow field  $F$  must still be discretised in order to evaluate the righthand side of equation (9), a cartesian grid (consisting of rectangular panels) is sufficient for this purpose, and this grid need not be fitted to the shape of the aerofoil contours. In fact, this grid need only be generated where the transonic terms  $\sigma_s$  and  $\sigma_u$  are significant, i.e. near the aerofoil (see [5],[15]). Also, both the steady and unsteady calculations can take place on the same grid.

### 3. PROPOSED NUMERICAL SCHEME - AN OUTLINE

The numerical scheme proposed here involves the two stages described above, namely, initial calculation of the steady flow field, followed by calculation of the unsteady flow field. Each stage uses the same grid, but with differing Green's functions and  $\sigma$  terms. The calculation scheme for each stage is the same and proceeds as follows:

**Step 0** Evaluate  $\phi = \phi_C$  on the aerofoil contour using a normal (subsonic) panel method, that is, setting the field source strength  $\sigma$  to zero. The aerofoil surface is divided into panels and the unknown  $\phi_C$  is approximated in some fashion, e.g. as being constant on each panel. The known quantity  $\frac{\partial \phi}{\partial N}$  is also approximated in some consistent fashion. The equation for  $\phi_C$ ,

$$E\phi_C = \int_C (G \frac{\partial \phi}{\partial N} - \phi_C \frac{\partial G}{\partial N}) d\ell \quad (13)$$

where  $C = C_S \cup C_W$  becomes a matrix equation for the discrete  $\phi_C$  values.

**Step 1** Evaluate  $\phi = \phi_F$  in the flow field  $F$  using the values of  $\phi_C$  calculated at the previous step, and the current value of  $\sigma$ . The flow field is discretised into panels on which, for example,  $\phi_F$  is constant. The discrete version of

$$E\phi_F = \int_C (G \frac{\partial \phi}{\partial N} - \phi_C \frac{\partial G}{\partial N}) d\ell + \int_F G \sigma dS \quad (14)$$

yields  $\phi_F$  at each field panel.

**Step 2** Evaluate the field source strength  $\sigma$  by applying finite differences to formulas (4) or (8). The expression for  $\sigma$  is non-linear for the steady problem, while for the unsteady problem it is linear but depends on the calculated steady solution.

**Exit** iteration loop if field source strengths have converged.

**Step 3** Evaluate  $\phi = \phi_C$  on the aerofoil boundary using the transonic panel method (with field source term). This step echoes Step 0 and involves solution of the linear matrix equation arising from discretisation of

$$E\phi_C = \int_C (G \frac{\partial \phi}{\partial N} - \phi_C \frac{\partial G}{\partial N}) d\ell + \int_F G \sigma dS \quad (15)$$

**Go to Step 1**



Stability and convergence have not been discussed here and are not guaranteed. However, in [15] encouraging results are given for the convergence of a similar natural iteration scheme. For subcritical (shock-free) flow convergence to acceptable accuracy occurs within 5 iterations, while for supercritical flows (with shocks) convergence occurs within 30 iterations.

#### 4. SUBSONIC PROBLEM

The subsonic problem is a subset of the transonic problem, corresponding to Step 0 above, in which the field source strength is zero and the steady Green's function (11) is used. This problem is considered first in order to identify areas of possible difficulty in the panelling and/or evaluation of the influence coefficients, and to provide the programming framework for later extensions to steady transonic flow (involving non-linear terms and iteration) and unsteady flow (more complicated Green's function).

The appropriate boundary value problem for the steady potential  $\phi$  is to satisfy equation (2) with associated boundary conditions

$$\frac{\partial \phi}{\partial n} = -\mathbf{i} \cdot \mathbf{n}. \quad (16)$$

on the aerofoil boundary. As seen already, the Prandtl-Glauert transformation puts the transonic small perturbation equation into the form given in equations (3) and (4), while the boundary conditions (16) become

$$\frac{\partial \phi}{\partial N} = -\frac{\mathbf{i} \cdot \mathbf{N}}{\beta^2}. \quad (17)$$

Note that  $\mathbf{N}$  is the normal to the aerofoil in the transformed flow domain, and that higher order terms have been neglected here, consistent with the linearisation of the governing equation.

The integral equation (13) for  $\phi$ , obtained by application of Green's formula, is solved by the method of collocation. The aerofoil contour  $C_S$  is assumed to consist of  $N_t$  discrete panels  $P_j$  whose end points are  $(x_{j-1}, y_{j-1})$  and  $(x_j, y_j)$ , with equation

$$x = a_j y + b_j. \quad (18)$$

(In [2] the wing surface is fitted exactly and the boundary integrals are carried out numerically, whereas in the Morino method an approximate geometry is used and the integrals are usually calculated analytically, especially near singularities.) The collocation points  $(\xi_i, \eta_i)$  are taken to be the  $N_t$  panel midpoints, with associated values of the potential  $\phi_i = \phi(\xi_i, \eta_i)$ .

At the  $i$ th collocation point we have

$$\pi \phi_i = \int_{C_S} \log R_i \frac{\partial \phi}{\partial N} d\ell - \int_{C_S} \phi \frac{\partial}{\partial N} (\log R_i) d\ell - \int_{C_W} \phi \frac{\partial}{\partial N} (\log R_i) d\ell \quad (19)$$

where  $R_i = \sqrt{(x - \xi_i)^2 + (y - \eta_i)^2}$ .

Assuming  $\phi$  is constant on each panel, and treating the wake boundary as a straight line emanating from the trailing edge across which there is no pressure jump, we have

$$\begin{aligned}\pi\phi &= \sum_j \int_{P_j} \log R_i \left( -\frac{\mathbf{i} \cdot \mathbf{N}}{\beta^2} \right) d\ell \\ &\quad - \sum_j \phi_j \int_{P_j} \frac{\partial}{\partial N} (\log R_i) d\ell \\ &\quad - \Delta\phi_{te} \int_{CW} \frac{\partial}{\partial N} (\log R_i) d\ell\end{aligned}\quad (20)$$

for  $i = 1, 2, \dots, N_t$ , where  $\phi_{te}$  is the value of  $\phi$  at the trailing edge. This leads to

$$\begin{aligned}\pi\phi_i &= \sum_j \int_{P_j} \frac{1}{\beta^2} \log R_i dy \\ &\quad - \sum_j \phi_j \left[ - \int_{P_j} \frac{x - \xi_i}{R_i^2} dy + \int_{P_j} \frac{y - \eta_i}{R_i^2} dx \right] \\ &\quad - \Delta\phi_{te} \int_{x_{te}}^{\infty} \frac{y_{te} - \eta_i}{R_i^2} dx.\end{aligned}\quad (21)$$

for  $i = 1, 2, \dots, N_t$ , where  $(x_{te}, y_{te})$  are the coordinates of the trailing edge, and  $N_{te}$  is the panel number of the (upper) trailing edge panel. Evaluating the integrals we have, finally

$$\pi\phi_i = \sum_{j=1}^{N_t} B_{ij} - \sum_{j=1}^{N_t} A_{ij}\phi_j - \sum_{j=N_{te}}^{N_{te}+1} W_{ij}\phi_j \quad (22)$$

for  $i = 1, 2, \dots, N_t$ , where

$$\begin{aligned}B_{ij} &= \frac{1}{(a_j^2 + 1)\beta^2} \left[ (y - \eta_i + a_j(x - \xi_i))(\log R_i - 1) \right. \\ &\quad \left. + (a_j\eta_i + b_j - \xi_i) \arctan\left(\frac{y - \eta_i + a_j(x - \xi_i)}{a_j\eta_i + b_j - \xi_i}\right) \right]_{y_{j-1}}^{y_j},\end{aligned}\quad (23)$$

$$A_{ij} = \begin{cases} \left[ -\arctan\left(\frac{y - \eta_i + a_j(x - \xi_i)}{a_j\eta_i + b_j - \xi_i}\right) \right]_{y_{j-1}}^{y_j} & \text{for } a_j \neq \infty \\ \left[ \arctan\left(\frac{x - \xi_i}{y_j - \eta_i}\right) \right]_{x_{j-1}}^{x_j} & \text{for } a_j = \infty, \end{cases} \quad (24)$$

and

$$W_{ij} = \pm \lim_{X \rightarrow \infty} \left[ \arctan\left(\frac{X - \xi_i}{y_{te} - \eta_i}\right) - \arctan\left(\frac{x_{te} - \xi_i}{y_{te} - \eta_i}\right) \right], \quad j = \{N_{te}+1\} \quad (25)$$

The above set of simultaneous equations is now solved for the unknowns  $\phi_j$ .

In the unsteady case, the Green's function is so complicated that the influence coefficients can only be evaluated numerically. Numerical experiments have

determined that for the steady problem a four point Gaussian rule is sufficiently accurate for use in evaluation of the influence coefficients, except near the collocation point where it becomes necessary to extract the singularity and evaluate that part separately (analytically).

For interfering aerofoils/components the wake terms  $W_{ij}$  need very careful treatment, especially when the idealised (i.e. straight) wake of one aerofoil component intersects with another component downstream. An interpretation used successfully here is that the arctan terms in  $W_{ij}$  represent the angles  $\beta$  and  $\alpha$  respectively, shown in Figure 2. These angles change continuously as the point  $(\xi_i, \eta_i)$  crosses the wake (even though the arctan function jumps). As a result,  $\pm 2\pi$  must be added to  $W_{ij}$  where necessary to retain continuity.

Once the potential has been obtained, an obvious method for calculating the pressure coefficient would be to differentiate equation (22) with respect to  $\xi_i$  and  $\eta_i$  and use these derivatives in all calculation. This works quite well within the flow field (provided the factor of  $\pi$  is replaced by  $2\pi$  in (22)). However, it also leads to large errors near the aerofoil because of the assumption that the potential varies as a step function. The function given by the right hand side of equation (22) becomes almost step-like between collocation points, causing the calculated derivatives to be useless near these points, i.e. on the aerofoil boundary.

Since direct calculation is impossible, an alternative is to calculate derivatives by finite differences. This is done using the collocation points, at which  $\phi$  is known best.

Returning to the original unscaled co-ordinates, the pressure coefficient  $C_p$  is calculated from the following formula, applicable to thin geometries:

$$C_p = -2[\mathbf{i} \cdot \mathbf{q} + \frac{1}{2}\mathbf{q} \cdot \mathbf{q}] + M^2(\mathbf{i} \cdot \mathbf{q})^2 \quad (26)$$

where  $\mathbf{q} = \nabla\phi$  is the perturbation velocity. On the aerofoil boundary we write

$$\mathbf{q} = q_s \mathbf{s} + q_n \mathbf{n} \quad (27)$$

where  $\mathbf{s}$  and  $\mathbf{n}$  are the tangential and normal vectors respectively. The pressure coefficient can thus be rewritten as

$$C_p = -2[q_s(\mathbf{i} \cdot \mathbf{s}) + \frac{1}{2}(q_s^2 - q_n^2) - \frac{1}{2}M^2(q_s^2(\mathbf{i} \cdot \mathbf{s})^2 - 2q_s q_n^2(\mathbf{i} \cdot \mathbf{s}) + q_n^4)], \quad (28)$$

which has the advantage that the magnitude of the normal velocity component  $q_n = -\mathbf{i} \cdot \mathbf{n}$  is known exactly from the boundary conditions, while the magnitude of the tangential velocity  $q_s$  is easily approximated by finite differences at the boundary. Thus finally, the following formula applies for the pressure coefficient at the  $i$ th collocation point:

$$C_{pi} = -2 \left[ \frac{\Delta\phi_i \Delta\xi_i + \frac{1}{2}(\Delta\phi_i)^2 - \frac{1}{2}(\Delta\eta_i)^2}{(\Delta\xi_i)^2 + (\Delta\eta_i)^2} \right] + M^2 \left[ \frac{(\Delta\phi_i)^2(\Delta\xi_i)^2 - 2\Delta\phi_i \Delta\xi_i (\Delta\eta_i)^2 + (\Delta\eta_i)^4}{[(\Delta\xi_i)^2 + (\Delta\eta_i)^2]^2} \right] \quad (29)$$

where

$$\Delta\phi_i = \phi_{i+1} - \phi_{i-1}, \quad (30)$$

$$\Delta\eta_i = \eta_{i+1} - \eta_{i-1} \quad (31)$$

and

$$\Delta\xi_i = \xi_{i+1} - \xi_{i-1}. \quad (32)$$

## 5. RESULTS AND DISCUSSION

Results have been computed for the case  $M = 0$  for three test configurations, namely

1. A circle with unit radius centered at the origin.
2. A NACA0012 aerofoil at zero angle of attack.
3. Williams configuration B [16] (aerofoil plus flap at 10 deg).

Figures 3 to 5 are plots of  $-C_p$  vs chordwise position for these three configurations. The exact incompressible solutions are denoted by lines and numerical solutions are denoted by asterisks.

The agreement in all cases is excellent. The agreement at the trailing edges in configuration 3 (Figure 5) is surprisingly good in view of the assumption that each wake is a straight line emanating from the trailing edge.

## 6. CONCLUSION

In this article the outline of a numerical scheme for the calculation of two-dimensional unsteady transonic aerodynamics has been presented. Excellent results have been obtained for the first step towards implementing this scheme, that is, for the special case of steady subsonic aerodynamics.

Despite the apparent simplicity of the outlined scheme, implementation for unsteady and transonic aerodynamics will still involve significant leaps in difficulty. For unsteady flow, a more complicated Green's function will mean that the integrals involved can no longer be evaluated in closed form. Difficulties are also anticipated in the application of the Kutta condition at the trailing edge. For transonic flow the addition of field source terms, and the consequent need for an iterative numerical scheme, will lead to demands on computer time and storage. Problems with stability and convergence of the numerical scheme are also a possibility.

## REFERENCES

- [1] J. D. Cole, *Modern developments in transonic flow*, SIAM J. Appl. Math., Vol. 29, No. 4, pp. 763-787, December 1975.
- [2] G. F. Fitz-Gerald, *Subsonic aerodynamics for complex configurations using the Green's function method*, RMIT-Dept-Mathematics-TR-3-1988, March 1988.
- [3] J. A. Gear, *Integral equation formulation for three-dimensional unsteady transonic flow*, Structures Report 416, Aeronautical Research Laboratories, February 1985.
- [4] M. H. L. Hounjet, *Transonic panel method to determine loads on oscillating airfoils with shocks*, AIAA J., Vol. 19, No. 5, May 1981.
- [5] M. H. L. Hounjet, *A field panel method for the calculation of inviscid transonic flow about thin oscillating airfoils with shocks*, NLR-MP-81043, October 1981.
- [6] E. B. Klunker, *Contribution to methods for calculating the flow about thin lifting wings at transonic speeds - analytic expressions for the far field*, NASA-TN-D-6530, November 1971.
- [7] M. T. Landahl, *Unsteady Transonic Flow*, Pergamon Press, New York, 1961.
- [8] L. Morino, *A general theory of unsteady compressible potential aerodynamics*, NASA-CR-2464, December 1974.
- [9] L. Morino & C. -C. Kuo, *Subsonic potential aerodynamics for complex configurations*, AIAA J., Vol. 12, No. 2, pp. 191-197, February 1974.
- [10] L. Morino, L. -T. Chen & E. O. Sucio, *Steady and oscillatory subsonic and supersonic aerodynamics around complex configurations*, AIAA J., Vol. 13, No. 3, pp. 368-374, March 1975.
- [11] D. Nixon, *Two-dimensional aerofoil oscillating at low frequencies in high subsonic flow*, ARC-CP-1285, January 1973.
- [12] W. J. Piers & J. W. Sloof, *Calculation of transonic flow by means of a shock-capturing field panel method*, NLR-MP-79022U, July 1979.
- [13] H. Tijdeman, *Investigation of the transonic flow around oscillating airfoils*, NLR-TR-77090U, December 1977.
- [14] K. Tseng & L. Morino, *Nonlinear Green's function method for unsteady transonic flows*, in *Transonic Aerodynamics* (ed. D. Nixon), AIAA New York, pp. 565-603, 1982.

- [15] P. M. Sinclair, *An exact integral (field panel) method for the calculation of two-dimensional transonic potential flow around complex configurations*, Aeronaut. J., Vol. 90, No. 896, pp. 227-336, June-July 1986.
- [16] B. R. Williams, *An exact test case for the plane potential flow about two adjacent lifting aerofoils*, ARC Reports & Memoranda No. 3717, September 1971.

# FIGURES

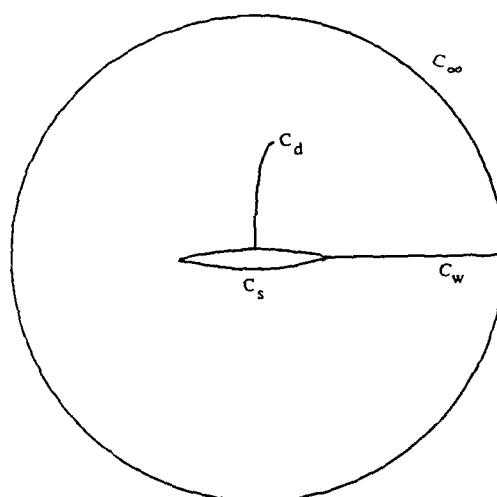


Figure 1. Flow domain showing important contours.

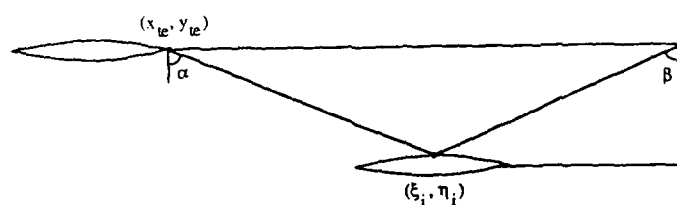


Figure 2. Diagram showing angles between the trailing edge, the wake and collocation point.

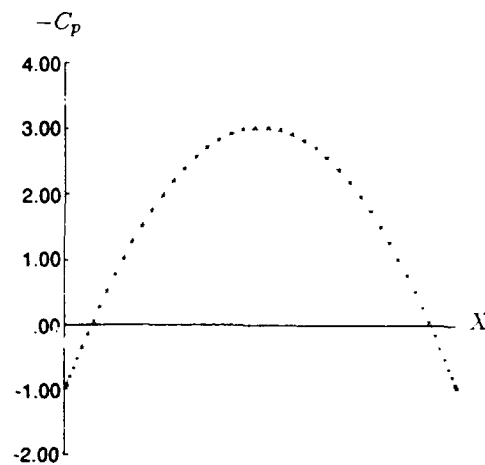


Figure 3.  $-C_p$  vs  $X$  for flow around a circular cylinder - Comparison of numerical results \* with the exact solution — .

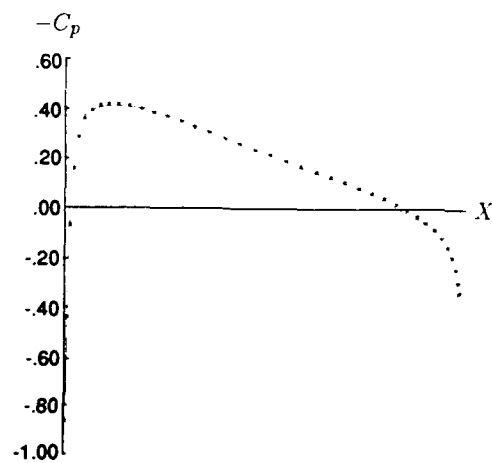


Figure 4.  $-C_p$  vs  $X$  for flow around a NACA0012 aerofoil at zero incidence - Comparison of numerical results \* with the exact solution — .



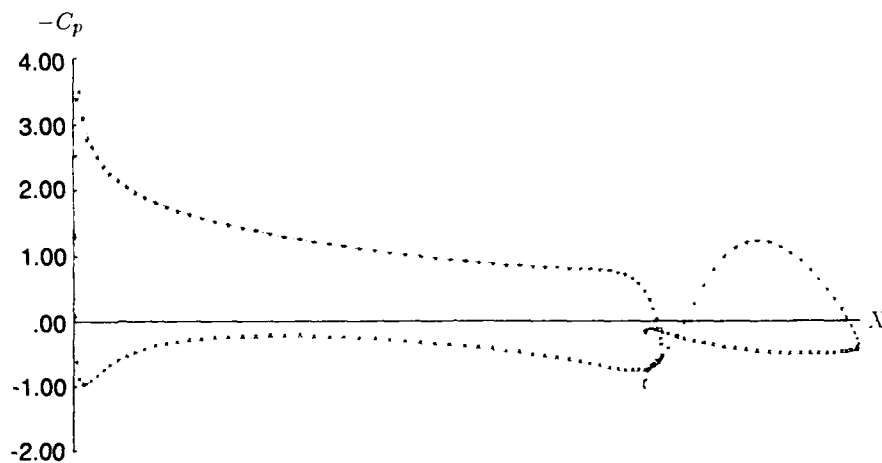


Figure 5.  $-C_p$  vs  $X$  for flow around Williams configuration B - Comparison of numerical results \* with the exact solution — .

## DISTRIBUTION

### AUSTRALIA

#### Department of Defence

##### Defence Central

Chief Defence Scientist )  
AS, Science Corporate Management ) shared copy  
FAS Science Policy )  
Director, Departmental Publications  
Counsellor, Defence Science, London (Doc Data sheet only)  
Counsellor, Defence Science, Washington (Doc Data sheet only)  
Scientific Adviser, Defence Central  
OIC TRS, Defence Central Library  
Document Exchange Centre, DSTIC (8 copies)  
Defence Intelligence Organisation  
Librarian H Block, Victoria Barracks, Melbourne (Doc Data sheet only)

##### Aeronautical Research Laboratory

Director  
Library  
Chief Aircraft Structures Division  
Divisional File - Aircraft Structures  
Author: I.H. Grundy

##### Materials Research Laboratory

Director/Library

##### Defence Science & Technology Organisation - Salisbury

Library

##### Navy Office

Navy Scientific Adviser (3 copies Doc Data sheet only)

##### Army Office

Scientific Adviser - Army (Doc Data sheet only)

##### Air Force Office

Air Force Scientific Adviser  
Engineering Branch Library

##### Universities and Colleges

Adelaide  
Barr Smith Library

Melbourne  
Engineering Library

Monash  
Hargrave Library

Sydney  
Engineering Library

NSW  
Physical Sciences Library  
Library, Australian Defence Force Academy

RMIT  
Library

SPARES (10 COPIES)

TOTAL (39 COPIES)

## DOCUMENT CONTROL DATA

PAGE CLASSIFICATION  
UNCLASSIFIED

PRIVACY MARKING

1a. AR NUMBER AR-006-091	1b. ESTABLISHMENT NUMBER ARL-STRUC-TM-527	2. DOCUMENT DATE AUGUST 1991	3. TASK NUMBER DST 90/033
4. TITLE AN OUTLINE OF A NUMERICAL SCHEME FOR CALCULATING TWO-DIMENSIONAL TIME LINEARISED TRANSONIC FLOW USING THE GREEN'S FUNCTION METHOD		5. SECURITY CLASSIFICATION (PLACE APPROPRIATE CLASSIFICATION IN BOXES) (E. SECRET (S), CONF. (C))  RESTRICTED (R), UNCLASSIFIED (U).  <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;">U</div> <div style="border: 1px solid black; padding: 2px;">U</div> <div style="border: 1px solid black; padding: 2px;">U</div> </div> DOCUMENT      TITLE      ABSTRACT	6. NO. PAGES  15  7. NO. REFS.  16
8. AUTHOR(S)  I. H. GRUNDY		9. DOWNGRADING/DELIMITING INSTRUCTIONS  Not applicable	
10. CORPORATE AUTHOR AND ADDRESS  AERONAUTICAL RESEARCH LABORATORY 506 LORIMER STREET FISHERMENS BEND VIC 3207		11. OFFICE/POSITION RESPONSIBLE FOR: SPONSOR <u>DSTO</u> SECURITY <u>-</u> DOWNGRADING <u>-</u> APPROVAL <u>CSTD</u>	
12. SECONDARY DISTRIBUTION (OF THIS DOCUMENT)  Approved for public release  OVERSEAS ENQUIRIES OUTSIDE STATED LIMITATIONS SHOULD BE REFERRED THROUGH DSTIC, ADMINISTRATIVE SERVICES BRANCH, DEPARTMENT OF DEFENCE, ANZAC PARK WEST OFFICES, ACT 2601			
13a. THIS DOCUMENT MAY BE ANNOUNCED IN CATALOGUES AND AWARENESS SERVICES AVAILABLE TO . . .  No limitations			
13b. CITATION FOR OTHER PURPOSES (IE. CASUAL ANNOUNCEMENT) MAY BE <input checked="" type="checkbox"/> UNRESTRICTED OR <input type="checkbox"/> AS FOR 13a.			
14. DESCRIPTORS  Two dimensional flow Transonic flow Green's function Steady flow			15. DISCAT SUBJECT CATEGORIES  2004
16. ABSTRACT  <i>A numerical scheme is outlined for the calculation of two dimensional time-linearised transonic flow about an aerofoil or collection of components, using the Green's function method. Computed results are presented for various configurations for the simpler sub-problem of steady subsonic flow.</i>			

PAGE CLASSIFICATION  
UNCLASSIFIED

PRIVACY MARKING

THIS PAGE IS TO BE USED TO RECORD INFORMATION WHICH IS REQUIRED BY THE ESTABLISHMENT FOR ITS OWN USE BUT WHICH WILL NOT BE ADDED TO THE DISTIS DATA UNLESS SPECIFICALLY REQUESTED.

16. ABSTRACT (CONT).

17. IMPRINT

**AERONAUTICAL RESEARCH LABORATORY, MELBOURNE**

18. DOCUMENT SERIES AND NUMBER

**AIRCRAFT STRUCTURES  
TECHNICAL MEMORANDUM 527**

19. COST CODE

**23211A**

20. TYPE OF REPORT AND PERIOD COVERED

21. COMPUTER PROGRAMS USED

22. ESTABLISHMENT FILE REF.(S)

23. ADDITIONAL INFORMATION (AS REQUIRED)